

Negation, inquisitiveness, and negative polar questions*

Scott AnderBois (Scott_AnderBois@Brown.edu)
Workshop on Non-Canonical Questions and Interface Issues

Konstanz – February 2014

1 Introduction

Long-standing puzzle:

- (1a-1d) are predicted to be equivalent by most theories of question semantics (e.g. Hamblin (1973), Groenendijk & Stokhof (1984)) ...

- ...yet they do different things:

(1)	a.	Is John cooking a Mexican dish?	PosQ
	b.	Is John not cooking a Mexican dish?	LoNEGQ
	c.	Isn't John cooking a Mexican dish?	HiNEGQ
	d.	Isn't John not cooking a Mexican dish?	HiLoNEGQ

- These differences are only partially pragmatic:
 - Polar questions with fronted or 'high' negation like (1c-1d) are fundamentally different than the others.
 - Questions with 'low' negation, (1b), are more restricted than a corresponding question with no negation (1a).

Source of the puzzle:

- Double negation is vacuous in classical logics.
- In particular, $\{p, \neg p\}$ is equivalent to $\{\neg p, \neg\neg p\}$

A path forward?

- Unlike classical logics, double negation in inquisitive semantics is not vacuous!
- In addition to their informative contributions, disjunctions and existential quantifiers have an *inquisitive* contribution (Ciardelli et al. (2013) and references therein)

*A great many people have given valuable feedback on this work at various stages (see AnderBois (2011) for a detailed list). I would like to especially single out Donka Farkas, Judith Aissen, Adrian Brasoveanu, and Floris Roelofsen for their generosity in discussing the ideas and data here. All remaining shortcomings are of course my own.

- Double negation leaves truth-conditions untouched, but eliminates the inquisitive component of the formula to which it applies¹.

Using existing inquisitive semantic theories for this purpose, however, has two problems:

1. Not fine-grained enough to distinguish high and low negation.
2. Fails to capture the clear sense in which the questions in (1) all contribute the same main issue to the discourse.

This talk:

- Propose a 'two-tiered' inquisitive semantics which comprises two kinds of issues that sentences may contribute:

Main issue: A set of alternative(s) whose resolution is expected (roughly, a QUD in Ginzburg (1996)'s sense)²

Projected issue: A set of alternatives which is made salient as a potential QUD, but whose resolution is not necessarily expected.

- The main issue is constant across the questions (1), whereas the projected issue varies depending on the form of the question.
- Show that this semantics plus plausible pragmatics derives core properties of positive, low negative, and high negative polar questions in English: (1a), (1b), and (1c).

Road Map

§2 describes the properties of the three types of polar questions we are focused on;
§3 develops and motivates a 'two-tiered' inquisitive semantics;
§4 examines the pragmatic reasoning linking §2 and §3;
§5 concludes.

2 Positive and negative polar questions

- Based on their surface form, we distinguish three primary varieties of polar questions:

(2)	a.	Is John cooking a Mexican dish?	PosQ
	b.	Is John not cooking a Mexican dish?	LoNEGQ
	c.	Isn't John cooking a Mexican dish?	HiNEGQ

- NB. the terms 'low' and 'high' are not to be confused with the terms 'inner' and 'outer' found in previous literature (e.g. Ladd (1981), Romero & Han (2004)).
- I use the terms 'low' and 'high' as descriptive terms for the position of overt negation within the question.

¹Indeed, Groenendijk & Roelofsen (2009) define a non-inquisitive closure operator, $!$, in terms of double negation.

²While the basic conception is, of course, shared with Roberts (1996) as well, the current account is more like Ginzburg's in that we treat an assertion as introducing a single alternative as QUD (i.e. "Should we accept this proposal?").

2.1 Properties of PosQs

Weak positive bias

- PosQs often exhibit a (weak) bias towards the affirmative answer (e.g. Bolinger (1978), Büring & Gunlogson (2000), van Rooÿ & Šafářová (2003)):
- We assume following van Rooÿ & Šafářová (2003) that bias in polar questions is not purely epistemic, but rather a combination of epistemic and bouletic.

- (3) a. Is she left-handed?
b. Is she right-handed?

However, PosQs can also occur with no apparent bias (see Büring & Gunlogson (2000)):

- (4) **Scenario:** Questions on a fair exam.
a. Is [b] a fricative?
b. Is [s] a fricative?

Elliptical secondary responses

- PosQs also readily allow for elliptical ‘secondary answers’ such as the response in (5)
- (5) **Context:** We are organizing a potluck dinner and want to have cake, cookies, and pie as desserts.
a. Is John bringing a cake?
b. Yeah, chocolate.

Properties of PosQs:

- Often show positive bias, but are also felicitous in clearly neutral contexts.
- Readily allow for elliptical secondary answers.

2.2 Properties of LoNegQs

- Some previous have lumped together LoNegQs and (certain) HiNegQs as deserving the same analysis (e.g. Ladd (1981), van Rooÿ & Šafářová (2003))
 - However, Romero & Han (2004) point out that there are contexts where LoNegQs are felicitous while HiNegQs are not³:
- (6) **Scenario:** The speaker is organizing a party and she is in charge of supplying all the non-alcoholic beverages for teetotalers. The speaker is going through a list of people that are invited. She has no previous belief or expectation about their drinking habits. A says “Jane and Mary do not drink.”

³Romero & Han (2004) also give parallel data to these which include an NPI such as *either* in both versions and report the same judgments. That is, LoNegQs are different from HiNegQs with NPIs (whether or not these result from a semantic ambiguity).

- a. S: OK. What about John? Does he not drink?
b. #S: OK. What about John? Doesn't he drink?

- We follow Romero & Han (2004) in concluding that LoNegQs and HiNegQs are distinct⁴.
- LoNegQs can be used in cases where the speaker merely has a *hunch* or suspicion of a negative answer, while HiNegQs cannot.

Weak negative bias

- LoNegQs show a weak negative bias:
- (7) a. Does Billy not like chocolate cake?
b. Is that not cooked all the way through?
- Unlike PosQs, LoNegQs are infelicitous in cases where the speaker is neutral:
- (8) **Scenario:** Questions on a fair exam.
a. #Is [b] not a fricative?
b. #Is [s] not a fricative?

Further discussion expected

- Despite their negative bias, LoNegQs are infelicitous in contexts where the speaker expects the addressee to be able to give a simple negative response.
 - When the context supports the speaker's bias towards the negative answer, but a simple ‘yes’ or ‘no’ is expected, LoNegQs are infelicitous:
- (9) I didn't see Bill at the party. John, you greeted everyone who was at the party, and you have a perfect memory.
Q: #? Was Bill not there?
Q': Was Bill there?
- (10) **Scenario:** A strict vegan at a café asking a bakery owner, inquiring as to whether the focaccia (typically dairy-free) has dairy in it.
Q: Excuse me, I'm vegan. #Does your focaccia not have dairy in it?
Q': Excuse me, I'm vegan. Does your focaccia have dairy in it?

⁴It should be noted that some LoNegQs do have the feeling of HiNegQs. These often have an ‘archaic’ feeling or seem to reflect a particular oratorical style (consistent with Romero & Han (2004)'s discussion of the historical recency of negative preposing). We will set aside these ‘Gladiator LoNegQs’ in what follows:

- (i) **SCENARIO:** The gladiator protagonist, Maximus, effortlessly kills yet another competitor. The crowd reacts with stunned silence at Maximus' ruthless efficiency, rather than applause.
a. Maximus: Are you not entertained? Are you not entertained? Is this not why you are here?

- Note that this appears to be a fact about low negation itself, as POSQs with similar content are licit in this same context:

- (11) Q: Excuse me, I'm vegan. Is your focaccia vegan?
 Q': Excuse me, I'm vegan. Is your focaccia dairy-free?

Properties of LONEGQs:

- Consistent weak bias to negative answer.
- Convey expectation of extended discussion.

2.3 Properties of HiNegQs

- Previous literature has often regarded HiNEGQs as being ambiguous between ‘inner’ and ‘outer’ negation readings (Ladd (1981), Romero & Han (2004)).
 - However, the examples given consistently have NPIs such as *either* or other scalar terms like *even*.
 - While the licensing and contribution of these items with HiNEGQs is quite puzzling, we set aside these data in what follows.
 - To my knowledge, then, there is only a single example of a putative ‘inner’ HiNEGQ:
- (12) **Situation:** Bob is visiting Kathleen and Jeff in Chicago while attending CLS.
- Bob: I'd like to take you guys out to dinner while I'm here — we'd have time to go somewhere around here before the evening session tonight, don't you think?
 - Kathleen: I guess, but there's not really any place to go in Hyde Park.
 - Bob: **Oh, really, isn't there a vegetarian restaurant around here?**
 - Kathleen: No, about all we can get is hamburgers and souvlaki.
- Recalling van Rooÿ & Šafářová (2003)'s claim that polar question bias is not solely epistemic, however, this example is plausibly an ‘outer’ HiNEGQ.
 - Bob has a clear epistemic bias towards the negative answer.
 - His goal, however, is presumably to find a restaurant which accommodates the vegetarians in the group.
 - Therefore, Bob is plausibly still biased towards the positive answer.
 - Previous literature has identified two primary features of HiNEGQs: (a) Verum Focus, (b) a positive prior belief.

Verum Focus

- *Verum Focus* is the name given by Höhle (1992) for sentences like (13) which emphasize the truth-value of the asserted proposition.

- (13) a. Kimiko DID_F go to the Himalayas.
 b. It is {true/the case} that Kimiko went to the Himalayas.

- Romero & Han (2004) claim that HiNEGQs also exhibit verum focus, i.e. (14a) is better paraphrased by the examples in (14b) than (14c).

- (14) a. Isn't John baking a cake?
 b. Is it {true/the case} that John is baking a cake?
 c. Is John baking a cake?

Positive prior belief

- Ladd (1981) shows that HiNEGQs convey the speaker's prior belief/expectation that the positive (yes) answer was true.

- (15) a. A: Ok, now that Stephan has come, we are all here. Let's go!
 b. S: Isn't Jane coming?
 c. **Prior belief:** Jane is coming.

- (16) **Inference of Prior Belief (IPB):** HiNEGQs regularly convey that the questioner previously believed or expected that the positive answer was true.⁵

- One further aspect of HiNEGQs worth mentioning is their strong positive bias.
- We assume that this bias is fundamentally different than that of POSQs and LONEGQs, arising from the a more general dispreference for belief revision.

Properties of HiNEGQs:

- Emphasis on truth of the main issue.
- Positive prior belief (and concomitant default positive bias)

3 Two kinds of issues

- Previous literature examining different varieties of polar questions has focused primarily on the differences in *bias*.
- We have seen that POSQs, LONEGQs, and HiNEGQs also differ in a related way: what issues they steer the conversation towards, i.e. their *projected issues*.
 - POSQs promote discussion of certain sub-issues within the positive answer.
 - LONEGQs promote protracted discussion of evidence for negative answer

⁵There is one case where the prior belief conveyed is a negative one: HiNEGQs which also have low negation such as (1d).

- HiNEGQs discourage discussion of all issues beyond the main issue, i.e. emphasize the truth/falsity of the main issue.

Claim: these projected issues are rooted in different compositional semantics.

- Low and high negation interact with inquisitive content within the question radical (e.g. disjunction, indefinites, and covert existential quantification).

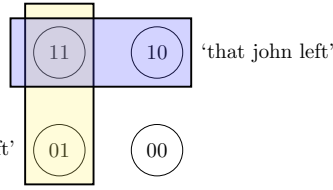
3.1 A single-tiered inquisitive semantics

- Core assumption: assertions, like questions, have possibly non-singleton sets of alternatives as part of their top-level meaning.
- One simple way to do this is to assign assertions the same sort of meanings as questions, sets of sets of possible worlds (e.g. Groenendijk & Roelofsen (2009)).
- For example, a disjunction like (18) will denote a set of two alternatives, as in (18a)⁶:

$$(17) \llbracket \varphi \vee \psi \rrbracket^{\mathcal{M},g,w} = \text{ALT}\{\alpha \subseteq W \mid \exists \beta \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w} : \alpha \subseteq \beta \text{ or } \exists \gamma \in \llbracket \psi \rrbracket^{\mathcal{M},g,w} : \alpha \subseteq \gamma\}$$

(18) John or Yesenia left.

- a. {that john left, that yesenia left }



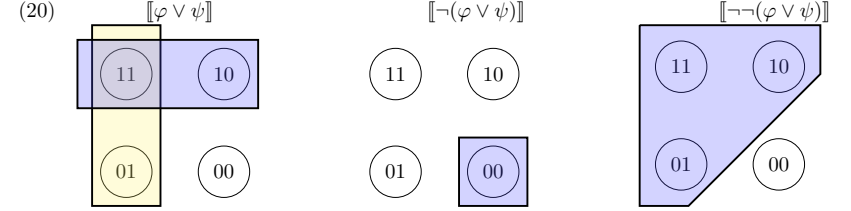
- This idea can be extended to existential quantification fairly straightforwardly for models with finite domains (e.g. AnderBois (2012), Ciardelli (2009))⁷.
- Since sentence meanings are sets of alternatives, sentential negation is defined as rejecting each alternative, as in (19).

$$(19) \llbracket \neg \varphi \rrbracket^{\mathcal{M},g,w} = \text{ALT}\{\alpha \subseteq W \mid \text{every } \beta \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w} \text{ is such that } \alpha \cap \beta = \emptyset\}$$

- It follows from this definition that *double negation* is no longer vacuous, as in (20).
- While it preserves truth-conditions, double negation eliminates inquisitive content, returning a set consisting of a single alternative in place of many.

⁶We can represent this denotation visually as in (18b), where circles represent possible worlds, numbers indicating the truth-values of two propositions in a given world, and boxes representing alternatives. ALT is Groenendijk & Roelofsen (2009)'s 'alternative closure' operator, which ensures that we get the *maximal* alternative meeting the stated condition.

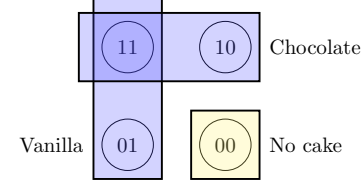
⁷AnderBois (t.a.) argues that slicing in English gives an empirical argument for an inquisitive semantics of this sort. Importantly, this argument extends beyond overt existential quantification to implicit quantification over not only individuals, but other domains such as event/state arguments.



- Since double negation is no longer vacuous, we predict subtly different meanings for positive and negative polar questions as in (21-22).

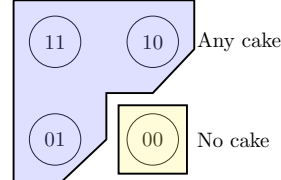
(21) a. Is John baking a cake? (POSQ)

b. $\llbracket \exists x.\text{bake}'(\text{john}, x) \vee \neg \exists x.\text{bake}'(\text{john}, x) \rrbracket$ POSQ



(22) a. Isn't John baking a cake? (HiNEGQ)

b. $\llbracket \neg \exists x.\text{bake}'(\text{john}, x) \vee \neg \neg \exists x.\text{bake}'(\text{john}, x) \rrbracket$ HiNEGQ



- The alternatives introduced by the indefinite *a cake* remain present in the POSQ, but not in the HiNEGQ.

Two problems:

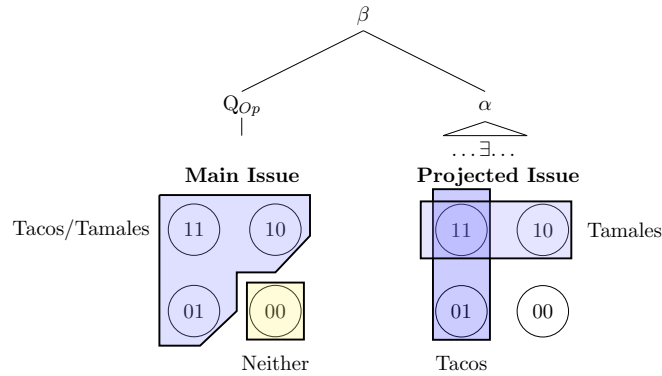
1. The sub-issue alternatives have the same status as the main issue ones in (21).
2. No clear way to capture the LoNEGQ's projected issue.

3.2 Distinguishing two kinds of ‘issues’

Both problems, we argue, can be resolved given a semantics which distinguishes two different kinds of issues: *main* issues and *projected* issues⁸.

- i.e. the ‘two-tiered’ interpretation of a formula φ , $\langle\langle\varphi\rangle\rangle$, will be an ordered pair $\langle\mathcal{M}, \mathcal{P}\rangle$ where both \mathcal{M} and \mathcal{P} are sets of alternatives.
- Pragmatically, these two kinds of issues typically have different effects:
 - The *main issue* establishes a new Question Under Discussion (QUD) in the sense of Ginzburg (1996).
 - The *projected issue* makes salient a potential QUD, but produces no obligation for the addressee to respond.
- Compositionally, these two issues arise (in English at least) from different sources:

- (23) a. Did Amelia bring a Mexican dish?
b.



- To do this, we will make use of a syntax where polarity is expressed across two positions, a high Σ_0 operator and a lower Neg_0 operator (Laka (1990) *et seq.*).
- In addition to these, we will assume a disjunctive Q_{op} of the expected form, i.e. $\lambda p.p \vee \neg p$ ⁹

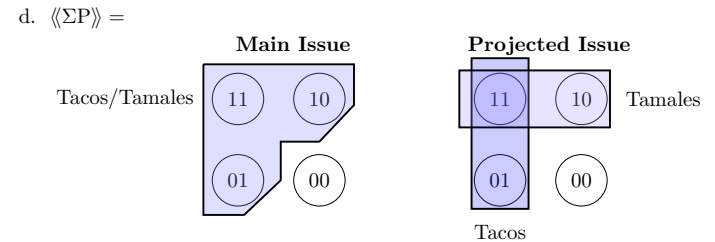
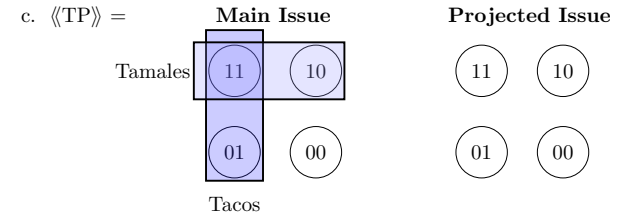
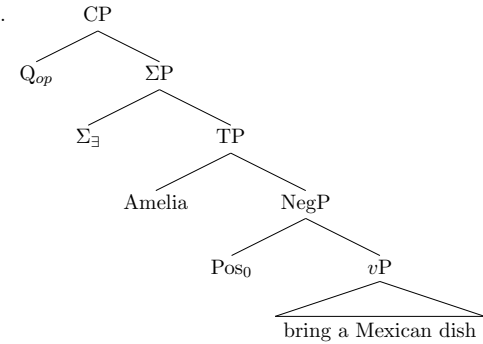
⁸See the Appendix for definitions of the operators we assume.

⁹Given that we complicate the treatment of negation, we actually must be more specific: it is high negation, Σ_{neg} , in the second disjunct.

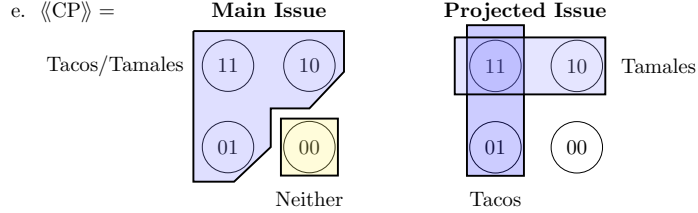
3.3 Positive polar questions

- Ignoring tense and other compositional details of the clause, the composition of POSQ is as in (24)
- The indefinite *a Mexican dish* introduces a set of alternatives which combines in pseudo-pointwise fashion.
- Σ_{\exists} provides existential closure over these alternatives.¹⁰
- Q_{op} adds in an alternative for the negation of the overt one.

- (24) a. Did Amelia bring a Mexican dish?
b.



¹⁰While this latter component is analogous to the existential closure proposed by Kratzer & Shimoyama (2002), the semantics as inquisitive in the sense that the alternatives are still part of the top-level meaning.



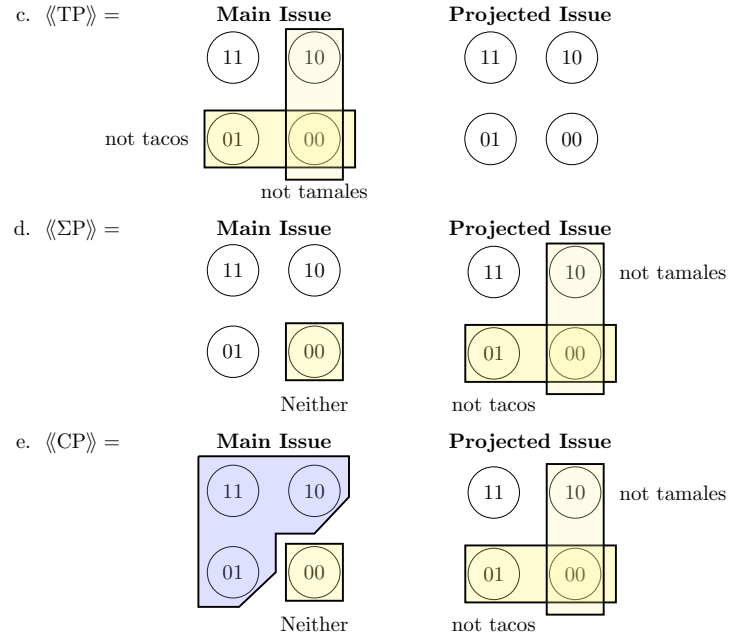
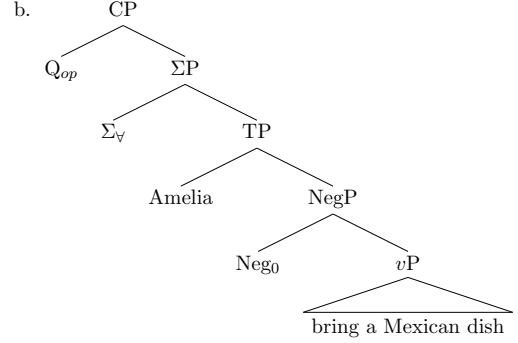
- $\langle\langle \Sigma P \rangle\rangle$ is what we take to be the meaning of the corresponding assertion.
 - Its main issue has a single alternative since this is the QUD it contributes, and it makes salient the alternatives corresponding to particular dishes.
- $\langle\langle CP \rangle\rangle$, then, succeeds in keeping separate the two kinds of issues in a way the single-tiered inquisitive semantics did not.
 - The main issue contains the same two alternatives (one positive, one negative) that negative polar questions will in following sections.
 - It is distinguished by highlighting a more fine-grained set of alternatives corresponding to different ways the positive answer may hold.
- Making this issue salient relates fairly directly to the licensing of elliptical secondary answers, ...
- ... and gives rise pragmatically to positive bias as we will discuss in §4

3.4 Low negation polar questions

- For LONEGQs, two things are different:
 - In place of Pos_0 , Neg_0 negates the alternatives in pseudo-pointwise fashion
 - Σ_V quantifies over these alternatives *universally*, rather than existentially.
- We leave open the question of what kind of relationship exists between Σ_0 and Neg_0 (e.g. agreement)¹¹

¹¹N.B. though that including Σ_{\exists} in place of Σ_V would in fact give a suitable interpretation for the wide-scope reading of the indefinite, at least in assertions.

(25) a. Did Amelia not bring a Mexican dish?



- Low negation, then, takes place in two steps: (i) creating a set of (pseudo-)pointwise alternatives, and (ii) universally quantifying over these.
- There is, then, an important asymmetry between the two-tiered semantics which emerges for negative and positive counterparts.

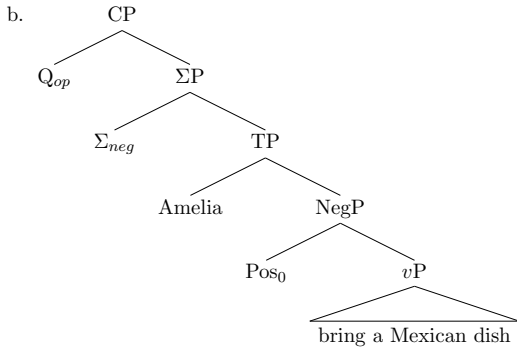
- In the positive version, (24), the projected alternatives in \mathcal{P} each entail a single main issue alternative in \mathcal{M} .
- In the negated sentence, (25), the direction of entailment is reversed.
- The projected issue is one which is logically prior to the main issue and is no longer relevant if the main proposal is resolved negatively.

- This is, we claim, a particular way to capture the intuition, discussed at length by Horn (1989), that negative sentences are in some sense ‘weaker’.
- The relationship between their projected and main issues means they do not push the conversation forward to the same extent.

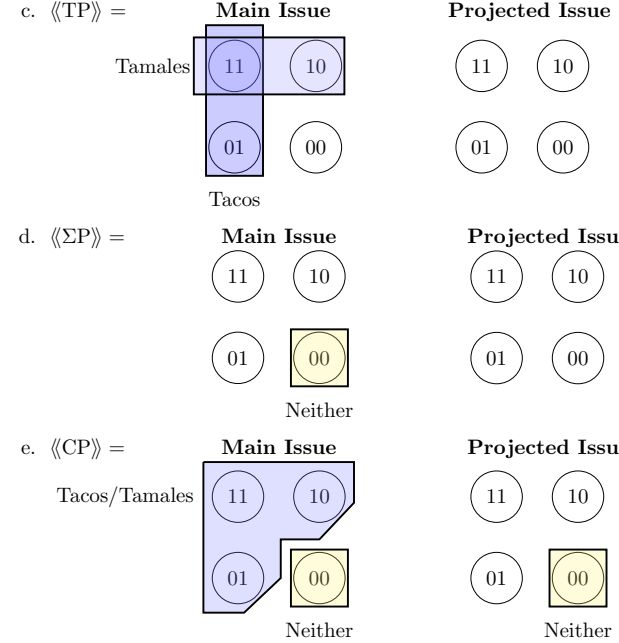
3.5 High negation polar questions

- In contrast to the two step process for low negation, we propose that high negation¹² performs both functions simultaneously.
- Crucially, therefore, the composition for high negation does not introduce a set of negated alternatives.
- As in Groenendijk & Roelofsen (2009), it simply returns the maximal alternative which doesn’t intersect any of the φ -alternatives.

(26) a. Didn’t Amelia bring a Mexican dish?



¹²While this I believe captures the intuition that HiNEGQs involve emphasis on truth, we leave open the question of whether all so-called Verum focus constructions deserve a unified analysis. There may well be several different ways in which the intuitive notion of ‘emphasis on truth value’ is realized.



- High negation eliminates fine-grained sub-alternatives, thereby giving greater emphasis to the truth/falsity of the main issue.
 - i.e. the lack of the further inquisitive content typical of POSQs and LONEGQs provides a way to capture “Verum focus”
 - At the same time, the semantics here has parallel’s with Reese (2007)’s proposal that HiNEGQs are simultaneously questions and assertions.

4 Pragmatic reasoning about polar questions

- As noted above, the two ‘tiers’ in our proposed semantics have pragmatic correlates:
 - QUDs for the main issue, \mathcal{M} .
 - Inquisitive issues/potential QUDs for the projected issue, \mathcal{P} .

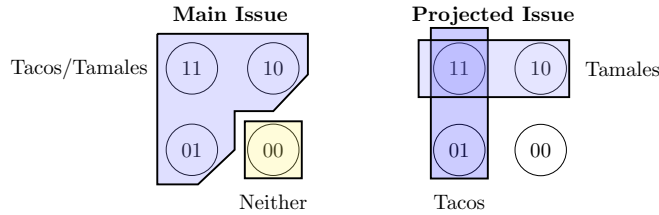
In this section, we briefly detail the pragmatic reasoning about these issues which we claim determines their distinctive properties.¹³

¹³While the semantics we have proposed here does differ in certain respects from AnderBois (2011), similar sorts of pragmatic reasoning are spelled out in more detail there.

4.1 PosQs

- The basic intuition across all three kinds of questions is that a cooperative speaker highlights potential QUDs for a reason, i.e. because they may be useful in the immediate future of the conversation.
- For a POSQ like (27), then, \mathcal{P} will only be useful in case the addressee provides a positive answer to \mathcal{M} .
 - Taking the speaker to cooperative, then, the addressee will conclude that the speaker must be biased towards the positive answer in this case.

(27) Did Amelia bring a Mexican dish?

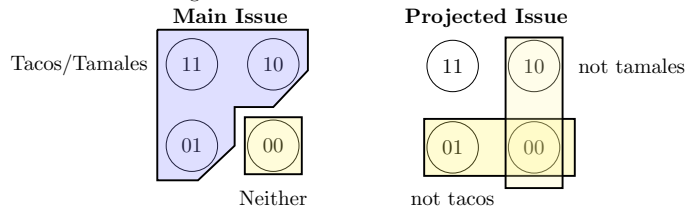


- However, a cooperative speaker may also choose the POSQ because it is the most unmarked form (i.e. best obeys the Maxim of Manner).

4.2 LoNegQs

- For LoNEGQs, however, \mathcal{P} is not a sub-issue of any alternative in \mathcal{M} .
- Therefore, a rational speaker ought to obey two conditions in using a LoNEGQ:
 1. The speaker is biased towards the negative answer (since the negated alternatives provide partial answers for it).
 2. The speaker believes that protracted discussion may be needed (since a negative answer would settle all alternatives in the projected issue).

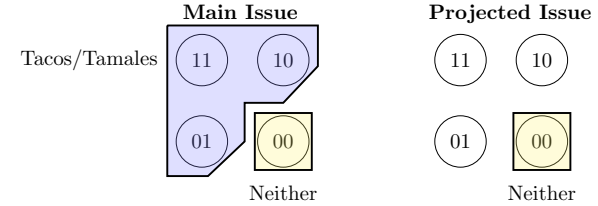
(28) Did Amelia not bring a Mexican dish?



4.3 HiNegQs

- The HiNEGQ on the other hand, uses a marked form in order to avoid highlighting any further issues.
- In essence, then, a speaker who utters a HiNEGQ:
 - Indicates they only want know the answer to main issue, ...
 - ... especially if it is negative.
- This, we claim, is the source of inference that the speaker previously believed the positive answer.
- The negative answer is of unusual importance since it would force the speaker to revise her prior belief.

(29) Didn't Amelia bring a Mexican dish?



5 Conclusions

- In this talk, we have presented some old and some new observations distinguishing POSQs, LoNEGQs, and HiNEGQs.
 - Beyond conveying bias, we have seen that different polar questions steer the conversation towards different addressing issues.
- We have proposed that these properties arise from pragmatic reasoning based on a semantics distinguishing two 'tiers' of issues.
- In addition to providing a principled account of negative polar questions, ...
- ... the proposed account is a step towards understanding the ways in which semantics – beyond questions themselves – can influence QUDs.

References

- AnderBois, Scott (2011) *Issues and Alternatives*. Ph.D. thesis, UC Santa Cruz.
- AnderBois, Scott (2012) Focus and Uninformativity in Yucatec Maya Questions. *Natural Language Semantics* 20(4): 349–390.
- AnderBois, Scott (t.a.) The semantics of sluicing: beyond truth-conditions. *Language* .
- Bolinger, Dwight (1978) Yes-no questions are not alternative questions. In *Questions*, 87–105.
- Büring, Daniel & Christine Gunlogson (2000) Aren’t positive and negative polar questions the same, ms.
- Ciardelli, Ivano (2009) Inquisitive Semantics and Intermediate Logics, mSc Thesis, University of Amsterdam.
- Ciardelli, Ivano, Jeroen Groenendijk, & Floris Roelofsen (2013) Inquisitive semantics: a new notion of meaning. *Language and Linguistics Compass* 7(9): 459–476.
- Ginzburg, Jonathan (1996) Dynamics and the semantics of dialogue. In *Language, Logic, and Computation, vol. 1*.
- Groenendijk, Jeroen & Floris Roelofsen (2009) Inquisitive Semantics and Pragmatics. In *Proceedings of SPR 09*.
- Groenendijk, Jeroen & Martin Stokhof (1984) *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph.D. thesis, University of Amsterdam.
- Höhle, Tilman (1992) Über Verum Fokus im Deutschen. In *Informationsstruktur und Grammatik*, 112–141.
- Horn, Laurence (1989) *A natural history of negation*. University of Chicago Press.
- Kratzer, Angelika & Junko Shimoyama (2002) Indeterminate Pronouns: The View From Japanese. In *The Proceedings of the Third Tokyo Conference on Psycholinguistics*, 1–25.
- Ladd, Robert (1981) A First Look at the Semantics and Pragmatics of Negative Questions and Tag Questions. In *CLS 17*, 164–171.
- Laka, Itziar (1990) *Negation in Syntax: on the nature of functional categories and projections*. Ph.D. thesis, MIT.
- Reese, Brian (2007) *Bias in questions*. Ph.D. thesis, University of Texas at Austin.
- Roberts, Craige (1996) Information Structure in Discourse. In *OSU Working Papers in Linguistics*, revised 1998 version, retrieved from author’s webpage 8/20/09.
- Romero, Maribel & Chung-Hye Han (2004) On Negative Yes/No Questions. *L & P* 27: 609–658.
- van Rooÿ, Robert & Maria Šafářová (2003) On polar questions. In *Proceedings of SALT 13*.

Appendix: a two-tiered inquisitive semantics

This appendix sketches a set of interpretive rules to compositionally provide two-tiered inquisitive denotations needed for POSQs, HiNEGQs, LoNEGQs, and corresponding assertions (see AnderBois (2011) for more detailed discussion of a related semantics).

- For ease of exposition, the definitions are given in terms of single-tiered inquisitive semantics (represented with straight double brackets, $\llbracket \varphi \rrbracket$).
- For a formula φ , its two-tiered interpretation, $\langle\langle \varphi \rangle\rangle$ is an ordered pair $\langle \mathcal{M}, \mathcal{P} \rangle$ where:
 - The main issue, \mathcal{M} , is a non-empty set of sets of possible worlds (i.e. a set of alternatives).
 - The projected issue, \mathcal{P} is a (possibly empty) set of sets of possible worlds (i.e. a set of alternatives).
- For readability, we will write such pairs in fractional notation: $\langle\langle \varphi \rangle\rangle = \frac{\mathcal{M}}{\mathcal{P}}$

Atomic Formulas:

$$\langle\langle \varphi \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \alpha \in \llbracket !\varphi \rrbracket^{\mathcal{M},g,w}\}}{\emptyset}$$

Low (pseudo-pointwise) Nego:

$$\langle\langle \text{neg}(\varphi) \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \text{there is some } \gamma \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w} \text{ s.t. } \alpha \cap \gamma = \emptyset\}}{\emptyset}$$

Disjunction: (\vee)

$$\langle\langle \varphi \vee \psi \rangle\rangle^{\mathcal{M},g,w} = \frac{\{\alpha \subseteq W \mid \alpha \in \llbracket !\varphi \rrbracket^{\mathcal{M},g,w} \text{ or } \alpha \in \llbracket !\psi \rrbracket^{\mathcal{M},g,w}\}}{\{\alpha \subseteq W \mid \text{there is some } \gamma \text{ in } \langle\langle \varphi \rangle\rangle \text{ or } \langle\langle \psi \rangle\rangle \text{ such that } \alpha \cap \gamma = \emptyset\}}$$

Existential alt-closure (Σ_{\exists}):

$$\langle\langle \Sigma_{\exists}(\varphi) \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \text{there is no } \beta \text{ s. t., for all } \gamma \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w}, \gamma \subseteq \beta, \text{ where } \alpha \cap \gamma = \emptyset\}}{\text{ALT}\{\alpha \subseteq W \mid \text{there is some } \gamma \in \langle\langle \varphi \rangle\rangle \text{ such that } \alpha = \gamma\}}$$

i.e.

$$\langle\langle \Sigma_{\exists}(\varphi) \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \text{there is some } \gamma \in \llbracket !\varphi \rrbracket \text{ such that } \alpha = \gamma\}}{\text{ALT}\{\alpha \subseteq W \mid \text{there is some } \gamma \in \langle\langle \varphi \rangle\rangle \text{ such that } \alpha = \gamma\}}$$

Universal alt-closure (Σ_{\forall}):

$$\langle\langle \Sigma_{\forall}(\varphi) \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \text{for all } \gamma \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w} : \alpha \subseteq \gamma\}}{\text{ALT}\{\alpha \subseteq W \mid \text{there is some } \gamma \in \langle\langle \varphi \rangle\rangle \text{ such that } \alpha = \gamma\}}$$

Negative alt-closure (Σ_{neg}):

$$\langle\langle \Sigma_{\text{neg}}(\varphi) \rangle\rangle^{\mathcal{M},g,w} = \frac{\text{ALT}\{\alpha \subseteq W \mid \text{for all } \gamma \in \llbracket \varphi \rrbracket^{\mathcal{M},g,w} : \alpha \cap \gamma = \emptyset\}}{\emptyset}$$

Question operator ($Q_{op}(\varphi)$): $\varphi \vee \Sigma_{\text{neg}}(\varphi)$